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Johannes Lorenz

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Adresse des Autors:

Johannes Lorenz
Wirtschaftswissenschaftliche Fakultät
Universität Passau
94030 Passau

Telefon: 0851 509 2443
Telefax: 0851 509 2442
E-Mail: johannes.lorenz@uni-passau.de

Population Dynamics of Tax Avoidance with Crowding Effects*

Johannes Lorenz†

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Abstract

There are two ways for taxpayers to avoid paying taxes: legally, through tax optimization and illegally, through tax evasion. The government reacts by altering the law, and by conducting audits, respectively. These phenomena are modeled as a population game, a strategic interaction between all taxpayers: the more taxpayers optimize, the lower the optimization result as a consequence of the government tightening the tax law. The more taxpayers evade, the higher the risk of detection because of the tax agencies increasing the audit probability. If the government reacts to changed optimization behavior with too large a delay, an equilibrium tax law cannot be reached. Tax codes should be updated rapidly in order to avoid a permanent change of the tax law, which is costly both for the legislator and the taxpayers facing legal uncertainty.

Keywords: tax avoidance, tax evasion, population games
JEL classification: C73, H26, K34

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†University of Passau, Innstraße 27, 94032 Passau, Germany (johannes.lorenz@uni-passau.de).

1. Introduction

In the tax game the main forces at play are taxpayers' efforts to avoid paying taxes and the tax authority's effort to enforce tax compliance. These forces act via two channels: first, the government needs to establish a tax code that regulates the details of liability for taxation and ensures horizontal and vertical equity. To reach this goal tax codes have to consider many real-world eventualities and grant tax exemptions where appropriate. Such tax shelters are used (and misused) by taxpayers. Also, since tax codes typically are complicated and to some extent inconsistent, taxpayers are able to legally avoid tax payments by searching for "loopholes". Second, taxpayers can illegally avoid taxes by simply not reporting their income to the tax authority or by reporting nonexistent expenses. The tax authority, in turn, conducts audits in order to detect and punish tax evaders.

This second channel has featured heavily in economic theory, starting with the seminal works of Allingham and Sandmo (1972) and Yitzhaki (1974) who regard tax evasion as a portfolio optimization approach, with the amount of evaded tax being the risky asset. One of their main findings is rather counterintuitive: the change in tax compliance after increasing tax rates can be positive. In fact, one would expect tax compliance to decrease with an increasing tax burden. Their work was extended to cover the tax authority's reaction and public goods provision by Cowell and Gordon (1988) who, amongst others, find that tax compliance may increase with increasing tax rates if public goods are over-provided. Reinganum and Wilde (1986) develop a game theoretical tax compliance model that assumes taxpayers with heterogenous income to play against the tax authority. The authors construct a separating equilibrium in which all taxpayers reduce their true income by a certain amount and the tax authority audits taxpayers with a certain probability which decreases with reported income. Erard and Feinstein (1994) extend the model by introducing a budget constraint for the tax authority and by assuming a certain fraction of taxpayers to be inherently honest. The complexity of the solution increases considerably. In equilibrium, both the taxpayers' income reports and the audit schedule depend on the taxpayers' income distribution. Since the model is not analytically solvable, the authors perform simulations. They find that the tax authority's net tax and penalty revenue rises rather slowly with an increasing share of honest taxpayers. Another string of literature relies on a principal-agent framework to analyze tax compliance issues. Assuming lump-sum taxes and penalties, Reinganum and Wilde (1985) find that random audit schedules where the audit probability is unconditional upon reported income are dominated by an audit policy in which the tax authority (the principal) sets a cutoff level, with lower reports are always audited and higher reports are never audited. They assume that the tax authority can commit to a certain schedule.[1]

With regard to the first channel mentioned above—legal tax optimization—the literature is less extensive, however. Mayshar (1991) generalizes the Allingham-Sandmo model by introducing the notion of a "tax technology", formally, a function which takes as arguments the tax base, the taxpayer's tax-shielding effort and a vector of tax instruments adopted by the tax authority and gives as output the tax payment. The function is kept general, the tax technology is thus a "black box", and the tax-shielding effort can be interpreted as any kind of legal or illegal measure the taxpayer might take in order to reduce their tax burden. Taxpayers choose labour effort and tax-shielding effort whereas the tax authority chooses (costly) tax instruments. The model aims at providing a framework capable of giving a cost-benefit analysis of administrative tax instruments. Slemrod (2001) specifies the tax technology by assuming a linear tariff. Tax avoidance is modeled as a reduction of the tax base by a certain amount at costs that depend on the true income and the extent of avoidance. The model's main focus lies on explaining behavioral responses to taxation, namely, the taxpayer's choice of labor supply and tax avoidance effort, and the interdependence between both. Cowell (1990) distinguishes between (illegal) tax evasion and (legal) tax sheltering. He argues that taxpayers will either evade *or* shelter parts of their income: since the tax sheltering function is publicly known, tax sheltering implicitly causes taxpayers to reveal information about their true income to the fiscal authority. It follows that there can be a complete polarization between shelterers and evaders, with the "rich" sheltering and the "poor" evading.[2]

The models mentioned above focus on individual taxpayer characteristics: the authors investigate on how taxpayers choose labor effort, tax avoidance effort, and, in case of tax evasion, the amount of tax evaded. As with tax evasion, the fiscal authority's reaction (i. e., audit probability) is based on the taxpayer's income report. As with tax avoidance, the government may choose certain tax instruments. However, the economic damage of both legal and

[1] See Andreoni, Erard, and Feinstein (1998) for an overview.

[2] See Slemrod and Yitzhaki (2002) for an overview.

illegal tax avoidance is heavily determined by the number of taxpayers applying such strategies, and so may be the tax authority's reaction. Formally, the tax agency's audit function (concerning tax evasion) and the "tax technology" function (concerning tax avoidance) may take as arguments the *number* of taxpayers applying either strategy. If so, an individual taxpayer's benefit from tax optimization or tax evasion is determined by the behavior of their fellow taxpayers rather than by the tax agency's reaction to an individual tax report: the game is no longer played between an individual taxpayer and the tax agency, but rather between *all* taxpayers.

Concerning tax evasion, this "crowding effect" can be motivated as follows: tax agencies, while auditing a set of tax reports, should be able to estimate the share of tax evaders based on the detection rate rather easily. It seems reasonable to assume that auditing activities are broadened if it turns out that there are many tax evaders, while auditing is cut down if the taxpayers turn out to be predominantly honest. The relationship is kept general in this paper, however, for lack of empirical evidence on its shape.

As with tax avoidance, there are several potential reasons for a "crowding effect". First, the government gets aware of the necessity to close loopholes not until they are exploited by a sufficiently large number of taxpayers. Also, if loopholes are used by a small fraction of taxpayers only, closing them is simply not profitable. However, governments have to intervene if too many taxpayers save on taxes in a way that is legal though not in the intention of the legislator. They do so by altering the tax law, or by adding additional legal norms. For example, most tax codes contain *thin-capitalisation rules* which limit the companies' possibilities of exploiting interest tax shields. Currently, the OECD "base erosion and profit shifting" (BEPS) project aims at prohibiting prominent structures like the "Double Irish with a Dutch Sandwich". Moreover, many tax authorities impose general *anti-tax-avoidance doctrines* which are not necessarily part of the tax law. They restrict tax avoidance directly by limiting the resulting tax savings.[3] Examples are the business purpose doctrine or the economic substance doctrine. Basically, such doctrines state that transactions will not be regarded by the tax law if their only purpose is a reduction of the tax liability. The presence of anti-tax-avoidance doctrines and legal norms that prohibit the usage of certain tax avoidance schemes reduces the taxpayers' profit from engaging in legal tax optimization. However, a tight tax law is also

[3] Weisbach (2002) discusses the efficiency of anti-tax-avoidance doctrines.

costly. Both the government and the taxpayers suffer bureaucracy costs from a high level of tax complexity. Also, multinational companies might refrain from investing in countries with tight tax laws. Hence, in part, countries could compete over the laxness of the tax law rather than over tax rates. Indeed, countries like the United States, Spain and Ireland recently weakened or abolished their thin capitalisation rules (Haufler & Runkel, 2012). This shows that governments basically have incentives to laxen their tax laws, if possible. In other words, if no one would optimize, governments would desire a tax law which is both uncomplicated and allows for generous tax savings. If such a lenient tax law is then heavily exploited by taxpayers, however, governments would need to tighten it again.[4] Another explanation for a crowding effect could be the government's aim to reach a certain budget target. For an example, consider a government's budget target of $ 90. Legal tax avoidance / tax optimization is interpreted as taxpayers applying for tax refunds. If ten taxpayers each pay taxes of $ 10 the total tax revenue exceeds the budget by $ 10. The excess amount is divided among the optimizing taxpayers. If only one taxpayer optimizes they receive the whole amount of $ 10. However, if five taxpayers optimize, each get a tax refund of $ 2.

Again, for lack of knowledge on the shape of the relation between the number of tax avoiders and the "profit" from tax avoidance, apart from being negative, it is kept general.

Strategic interactions where an individual plays against a whole society rather than a limited number of other individuals are referred to as population games (Hofbauer & Sigmund, 1998). Then, a Nash equilibrium is given not by a strategy choice of single individuals, but by population shares that each play pure strategies. In the basic model developed in this paper, taxpayers decide between two strategies, "optimization" and "non-optimization". "Optimization" involves tax planning costs that depend on the amount of pre-tax income. Tax

[4] This situation was modeled by Diller, Grottke, and Schneider (2013) as a single-shot two-player game between a taxpayer who can choose to exert a certain tax planning effort and a government which chooses a certain degree of tax complexity. Higher planning effort is associated with bigger tax savings but increasing planning costs. Higher complexity is associated with smaller tax savings and increasing complexity costs. Inter alia, the authors find that both planning effort and tax complexity increase with an increasing tax base. With regard to the results of the present article, it should be pointed out that Diller et al. (2013) find that an existing level of tax complexity and tax planning effort changes over time only if the costs of either tax complexity or tax planning change.

planning then leads to a certain tax refund. The size of the tax refund depends negatively on the share of optimizing taxpayers, as motivated above. There exists only one Nash equilibrium which involves a certain share of taxpayers who optimize while the remainder does not optimize. It turns out that the share of optimizing taxpayers increases with increasing tax rates, decreasing optimization costs and a generally higher optimization result. A dynamic version of the game is implemented by applying a pairwise proportional revision protocol. That is, when receiving an opportunity to update their strategy, a taxpayer meets another taxpayer at random and adopts their strategy with probability proportional to the payoff difference if the latter is positive. The evolution of the whole population's behavior can then be approximated by the replicator dynamic (Schlag, 1998), which was originally developed by Taylor and Jonker (1978) to capture the evolution of species by survival and reproduction of the fittest. Applying the replicator dynamic, it is shown that the Nash equilibrium is the only stable rest point and thus a good prediction for the outcome of the game. Since in reality the legislative process is rather slow, next, a delay is introduced to the tax law reaction function, which causes the population share of optimizing and non-optimizing taxpayers to oscillate over time. If the delay is small, the oscillation is dampened and over time the system approaches the Nash equilibrium. If the delay exceeds a certain threshold, however, the population state continues to oscillate. This result is especially interesting because of two points: first, it shows that the structure of the tax law changes "endogenously" without a change in institutional parameters like cost of tax complexity or cost of tax optimization. Second, looking only at the Nash equilibrium but neglecting the dynamic adaption process, the very result of an oscillating population state would not be found, giving a fundamentally different prediction for the outcome of the game. In reality, the process of legislative amendments is costly. Moreover, a permanently changing tax law creates legal uncertainty.[5] One potential policy implication is that the legislative process should be accelerated in order to reach an equilibrium and to avoid having to amend the law incessantly.

In the next step, the taxpayers are allowed to choose to evade taxes illegally as a third possible strategy. As explained above, in contrast to other tax evasion models, the audit probability is assumed to depend not on the tax return but on

[5] There is empirical evidence that that tax law uncertainty has a negative impact on investment (Edmiston, 2004).

the share of evading taxpayers within a population. Because of that, tax evasion is an all-or-none decision: taxpayers always report an income of zero once they choose the "evasion" strategy. Depending on parameter relations, two possible Nash equilibria can be identified. The first equilibrium requires that all three strategies are played by positive population shares. Then, the population share of optimizers is the same as in the two-strategy case, and the share of evaders increases to the detriment of the share of non-optimizing taxpayers if the penalty rate decreases or if the audit function generally decreases. As to the dynamic case, the Nash equilibrium again is the only stable rest point. If the parameters are chosen such that the payoff from non-optimization is lower than both the payoffs form tax optimization and tax evasion, the "non-optimization" strategy becomes extinct. Then, both increasing the tax rate and the penalty rate causes the share of optimizers to increase to the detriment of the share of tax evaders. Introducing a delay into the tax law reaction function causes the population state to oscillate over time, as in the two-strategy case. Again, if the delay is below a certain threshold, the Nash equilibrium is reached over time whereas if the delay is too large, the system keeps oscillating.

The remainder of this paper is structured as follows. In the next section a basic model with the two strategies "optimization" and "non-optimization" is presented; the game's equilibrium is characterized and a dynamic approach including a delayed government's reaction is outlined. Section 3 extends the model to allow for (tax) "evasion" as a third strategy. Again, the game is analyzed as a static and dynamic model. The paper closes with a brief summary.

2. Basic Model

2.1. Framework with Two Strategies

Consider a population of risk-neutral taxpayers as defined by the set $P = \{1, \ldots, N\}$. Every period, each taxpayer plays against the whole population of taxpayers . The set of strategies available to the taxpayers is denoted by $S = \{o, n\}$ where o denotes legal tax optimization and non-optimization is denoted by n. Let x_s denote the share of the population that chooses strategy $s \in S$. The population state is given by $X = \{x \in \mathbb{R}^2_+ : \sum_{s \in S} x_s = 1\}$. The reaction of the government is not considered explicitly. Instead its actions are reflected by the payoff functions. That is, it is assumed that the state of the tax law is a function of the number of agents who try to reduce their tax burden in a legal manner:

the more agents optimize, the less can be gained through optimization by an individual taxpayer.

Formally, tax savings as a fraction of the tax rate are denoted by a continuous, strictly decreasing function $o(x_o)$ with $o(1) = 0$ and $o(0) = 1$.[6] If all taxpayers optimize ($x_o = 1$), the tax code takes a state that allows for no more tax savings. By contrast, if no one optimizes ($x_o = 0$), the first taxpayer to optimize can reduce their tax rate to zero.[7] The payoff vector field $F : X \to \mathbb{R}^2$ consists of the following continuous payoff functions.

The payoff from tax optimization is given as

$$F_o(x) = y - \tau(1 - o(x_o))y - c(y), \tag{1}$$

where y is a taxpayer's income before tax, τ is the tax rate and $c(y)$ denotes the cost of tax optimization as a function of income. It seems reasonable to assume that $c'(y) > 0$ and $c''(y) \leq 0$: it is more expensive to "hide" higher income from the tax authority, the marginal tax planning costs decrease, however, because of economies of scale.[8] This can also be interpreted in the way that taxpayers with higher income are well-educated and thus find it easier to optimize taxes at the margin. Moreover, marginal optimization costs can also be constant with lower costs representing greater knowledge of the tax law, or higher capability. An optimizer can expect to receive the fraction $o(x_o)$ of their tax liability as a tax refund. As described above, the success of their optimization activities depends on the total number of optimizing agents. If a large number of taxpayers reduce their tax liabilities using loopholes in the law, the government will close

[6] A somehow comparable approach is presented by Weisbach (2002). He denotes the strenght of anti-avoidance doctrines by a parameter $\alpha \in [0, 1]$ where $\alpha = 0$ describes the absence of anti-avoidance doctrines and $\alpha = 1$ means that taxation cannot be avoided. $\alpha = 0$ does not necessarily imply that no taxes have to be paid, however, as does the corresponding case $o = 1$ in the present article.

[7] A systems-theoretical notion of tax compexity would suggest that $\frac{d}{dx_o}(1 - o(x_o)) > 0$ and $\frac{d^2}{dx_o^2}(1 - o(x_o)) < 0$, that is, tightening the tax law increases the government's tax revenue; however, the marginal tax revenue decreases because the most obvious loopholes are already closed. Referring to the tax savings function used in the present article, this would imply that $o'(x_o) < 0$ and $o''(x_o) > 0$. However, the results are valid without assuming that $o''(x_o) > 0$.

[8] Marginal tax planning costs are allowed to be constant in order to enable a simple linear cost structure. De facto, it is not necessary to make any assumptions on $c''(y)$.

these loopholes by adopting additional laws, thus complicating the tax code. Optimization activities will then be less successful.

Non-optimization delivers the payoff

$$F_n = y(1 - \tau), \tag{2}$$

which is certain and does not depend on the actions of other taxpayers. Note that optimizing may be never a beneficial strategy, even if only one taxpayer chooses "optimization" and hence $o(x_o) \to 1$. Still, it is possible that $y - c(y) < y(1 - \tau)$, i. e., the cost of tax optimization is higher than its benefit. Given the concave cost structure introduced above, this could happen for low values of y. A strategy which is never beneficial will become extinct in equilibrium. Since a trivial solution involving all taxpayers paying their taxes without optimization is of little interest, it is assumed that $c(y) < \tau y$ below; i. e., optimization costs are lower than the tax payment and thus tax optimization *can* (though need not always) be beneficial.

F is a potential game (Monderer & Shapley, 1996) since there exists a potential function $f : X \to \mathbb{R}$ which satisfies $\nabla f(x) = F(x) \forall x \in X$:

$$f(x) = x_n y(1 - \tau) + x_o(y(1 - \tau) - c(y)) + \tau y \int_0^{x_o} o(z)\, dz.$$

Since f is concave[9], all Nash equilibria are maximizers of f (see Sandholm, 2010, p. 60).

2.2. Equilibrium

The only Nash equilibrium of the game is given by the population state $\{x_o^\star, x_n^\star\}$ that satisfies the conditions

$$\tau o(x_o^\star) - \frac{c(y)}{y} = 0, \tag{3}$$

$$x_o^\star + x_n^\star = 1. \tag{4}$$

[9] The hessian $H_f(x) = \begin{pmatrix} \tau y o'(x) & 0 \\ 0 & 0 \end{pmatrix}$ has non-positive eigenvalues $\{\tau y o'(x), 0\}$.

Equation (3) is intuitive: the beneficial decrease in the tax rate due to optimization activities has to equate the cost of optimization relating to income. Note that in equilibrium the payoff of the strategy "optimization" is equal to the outcome of the strategy "non-optimization", $y(1-\tau)$. Hence, legal tax avoidance is not profitable. This is not surprising but rather a requirement for a Nash equilibrium: if in some population state x' one of the strategies is profitable, other taxpayers will adopt this strategy until there is no more excess return; the system converges to state $x^\star$.[10] Deriving (3) with respect to income y delivers

$$\frac{\partial x_o^\star}{\partial y} = \frac{c(y) - yc'(y)}{-\tau y^2 o'(x_o^\star)}, \tag{5}$$

which is positive as long as the average costs exceed marginal costs: $c(y)/y > c'(y)$. Making use of (3), the condition can be rewritten as $\tau o(x_o^\star) > c'(y)$: as long as marginal optimization costs are smaller than the optimization effect, the share of optimizing taxpayers will increase if income increases. If costs are assumed to be linear, the equilibrium share of optimizing taxpayers does not depend on the amount of income. If costs *decrease* with increasing income (possibly because better-educated individuals with higher income find it easier to shelter income from taxation), increasing income increases the share of optimizers. The same is true if costs are assumed to be constant.

The derivative of the equilibrium share of optimizing taxpayers $x_o^\star$ with respect to the tax rate,

$$\frac{\partial x_o^\star}{\partial \tau} = \frac{o(x_o^\star)}{-\tau o'(x_o^\star)}, \tag{6}$$

is unambiguously positive: higher tax rates cause the share of optimizing taxpayers to increase. This result is driven by the optimization costs which depend on income y only, but do not increase in the tax rate τ. As is immediately evident from (3), not surprisingly, fewer taxpayers optimize if they are confronted with a higher cost structure $c_H(y) > c(y) \forall y$. The share of optimizing taxpayers also decreases if they are confronted with a new tax law reaction function $o_L(x_o) < o(x_o) \forall x_o \neq 0, 1$ and vice versa.

[10] A dynamic adaption process of this kind is modeled explicitly in section 2.3.

In equilibrium the tax code will allow tax savings of

$$o(x_o^\star) = \frac{c(y)}{\tau y} \tag{7}$$

generating a total tax revenue of

$$\begin{aligned} T = &\sum_{i \in P: s=n} \tau y + \sum_{i \in P: s=o} \tau y (1 - o(x_o^\star)) \\ =& N \left(\tau y - x_o^\star c(y) \right), \end{aligned} \tag{8}$$

where N is the number of all taxpayers. An optimizing taxpayer's tax payment is given by $\tau y - c(y)$. The tax authority hence loses exactly the optimization cost $c(y)$ measured by the share of optimizing taxpayers.

2.3. Replicator Dynamics

The Nash equilibrium derived in the previous section is a static concept. So far, there is no explanation of how the equilibrium population state is actually reached. The taxpayers' behavior at the micro-level is modeled using the notion of a *revision protocol* (Sandholm, 2010, p. 121). Hereby, the evolutionary process for a number of N taxpayers is described by a Markov process. It is assumed that individuals receive opportunities to change their strategies at certain points in time. The time lags between the arrivals of revision opportunities are distributed independently according to an exponential distribution with rate λ. If a revision opportunity arrives, an individual taxpayer switches from strategy i to strategy j with probability r_{ij}/λ, where r_{ij} is called *conditional switch rate.* In this article the revision protocol known as *proportional imitation* will be applied, defining the conditional switch rate as $r_{ij} = x_j \left[F_j(x) - F_i(x) \right]_+$ (Schlag, 1998). For the two-strategy game introduced above the conditional switch rate from non-optimization to optimization is given by $r_{no} = x_o[F_o(x) - F_n]_+$. Intuitively, a taxpayer with the opportunity to switch strategies meets another taxpayer at random. With probability x_o they will meet an optimizer. If this is the case, the taxpayer will switch strategies only if the payoff from optimization exceeds the payoff from non-optmization. Given that, the taxpayer's probability of switching to the optimization strategy will be proportional to the payoff difference. This way of modeling the taxpayers' decision making has the pleasant feature that they need not be informed about properties like the population state x, the population's average payoff or other individuals' payoffs (except the payoff of the one taxpayer met at random). This is especially desirable in view of the fact that tax returns are undisclosed in most countries. Note that if no one optimizes, thus $x_o = 0$, the probability of switching to the optimization strategy is zero for all taxpayers. That is, strategies that are currently not played by a positive population share will not be invented under the proportional imitation protocol.

Given the pairwise proportional imitation revision protocol explained above the behavior of the Markov process can be approximated[11] by the dynamic

$$\begin{aligned} \dot{x}_o &= x_o \left(F_o(x) - x^T F(x) \right) \\ \dot{x}_n &= x_n \left(F_n - x^T F(x) \right), \end{aligned} \tag{9}$$

where $\dot{x}_i = \frac{d}{dt} x_i$ denotes the time derivative and x^T is the transposed vector of the population state. The system (9) is the well-known *replicator dynamic*. At a macro level the relative rate of change of a strategy is given by the difference between its own payoff and the mean payoff.

A fixed point is reached when the change over time is zero, hence $\dot{x}_o = \dot{x}_n = 0$ has to be fulfilled simultaneously. The replicator dynamic (9) features three fixed points

$$\begin{aligned} &\{x_n = 0, x_o = 1\}, \\ &\{x_n = 1, x_o = 0\}, \\ &\{x_n = x_n^\star, x_o = x_o^\star\}. \end{aligned}$$

[11] The expected number of revision opportunities arriving in the time interval $[0,\ dt]$ is given by $\lambda\, dt$. Thus, the number of revision opportunities received by non-optimizing agents in this time span is given by $N x_n \lambda\, dt$. The number of agents who switch to the optimization strategy (or continue to optimize) is given by $\frac{r_{no}}{\lambda} N x_n \lambda\, dt + \frac{r_{oo}}{\lambda} N x_o \lambda\, dt = N(x_n r_{no} + x_o r_{oo})\, dt$ and the number of agents who choose to become (or remain) non-optimizers is given by $N(x_o r_{on} + x_n r_{nn})\, dt$. The expected change in the population share of non-optimizing agents is given by $(x_o r_{on} - x_n r_{no})\, dt$. Eliminating the time differential one obtains the differential equation $\dot{x}_n = x_o r_{on} - x_n r_{no}$. Applying the pairwise imitation protocol one gets $\dot{x}_n = x_n(x_o[F_n - F_o(x)]_+ - x_o[F_o(x) - F_n]_+) = x_n(x_o F_n - x_o F_o(x))$ which can be rewritten as $\dot{x}_n = x_n(F_n - (x_n F_n + x_o F_o(x)))$. The derivation of the mean dynamic for a higher number of strategies is analoguous (Sandholm, 2010, p. 126).

The first two are corner solutions in which the whole population either optimizes or doesn't optimize, respectively. The third fixed point is the Nash equilibrium shown above. It is easily shown that the Nash equilibrium is the only stable fixed point of the system. Starting from $x_o = 0,\ \dot{x}_o = 0$, if a single taxpayer, for whatever reason, starts optimizing, the rate of change $\dot{x}_o$ becomes positive since

$$\left.\frac{\partial \dot{x}_o}{\partial x_o}\right|_{x_o=0} = \tau y - c(y) > 0$$

is positive by an assumption made earlier in this article. Thus, more taxpayers start optimizing until the Nash equilibrium is reached and one has $x_o = x_o^\star,\ \dot{x}_o = 0$. If the whole population optimizes, that is, $x_o = 1,\ \dot{x}_o = 0$ but one taxpayer decides to pay their taxes regularly, the rate of change $\dot{x}_o$ becomes negative since

$$\left.\frac{\partial \dot{x}_o}{\partial x_o}\right|_{x_o=1} = c(y) > 0.$$

Thus, more taxpayers refrain from optimizing and x_o decreases until the Nash equilibrium is reached again. The same kind of reasoning applies for the share of non-optimizing taxpayers since $x_n = 1 - x_o$. Figure 2.1 illustrates the situation. Over time the system converges to the Nash equilibrium from (almost) all initial population states. Only if *all* taxpayers optimize *(all* taxpayers don't optimize) the respective alternative strategy will never be "invented". As described above, however, these states are not robust to small perturbations. Figure 2.2 shows the evolution of the population state for different initial conditions. All illustrations below are plotted choosing the reaction functions and parameters stated in Table A.1 in Appendix A.

2.4. Delayed Amendments

In the previous section, the sequence of the game could be thought of as follows: first, all taxpayers simultaneously choose their respective strategies. The tax authority then observes the population state and chooses the state of the tax law. Third, taxes are collected and tax refunds are granted according to $o(x_o)$. It is, however, a strong assumption to demand that tax law be adjusted immediately

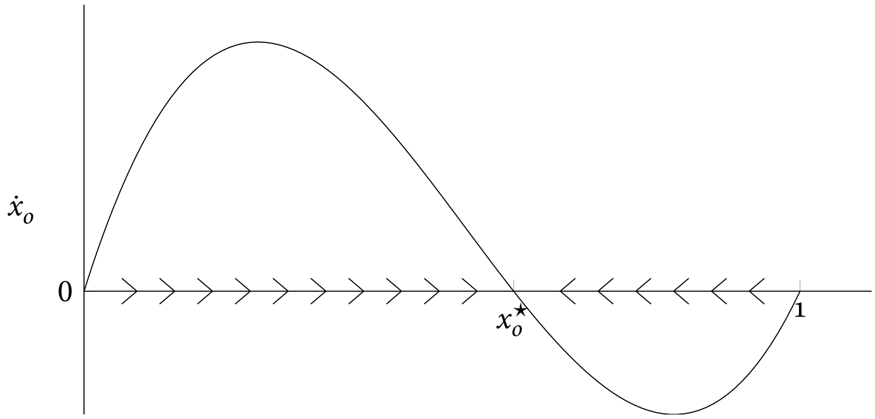

Figure 2.1: *Rate of change of the "optimization" strategy, $\dot{x}_o$, depending on the population share of optimizing taxpayers x_o.*

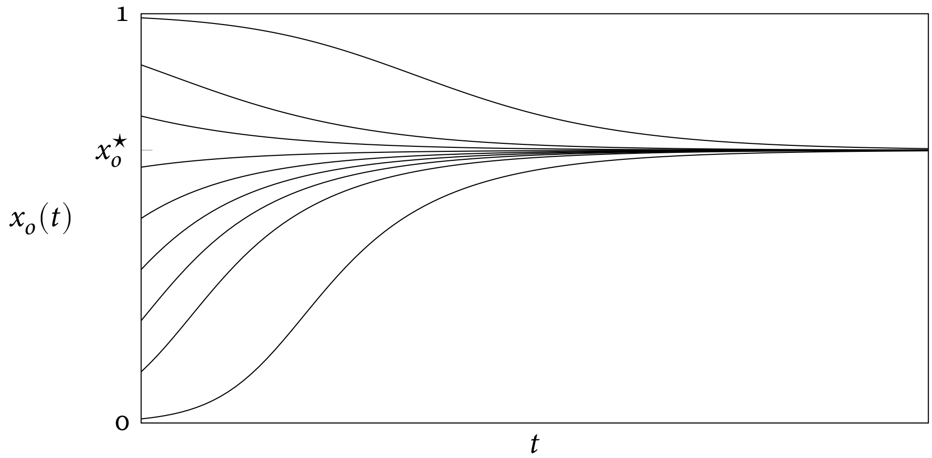

Figure 2.2: *Evolution of the share of optimizing taxpayers x_o over time for initial conditions varying from (almost) zero to (almost) one in steps of 0.125.*

depending on the number of optimizing taxpayers. It would be more realistic to assume that the tax authority amends the legislation in the subsequent period. In reality, however, amendments can take even longer. This phenomenon is captured by a delay parameter δ. δ is a positive real number which is interpreted as the time the tax authority needs to amend the tax code. A taxpayer deciding to optimize at time t hence receives a tax refund according to the tax code at time $t - \delta$. Denoting the population state as a function of time, the modified payoff vector field can be written as

$$\hat{F}(x(t)) = \begin{pmatrix} \hat{F}_o(x(t)) \\ \hat{F}_n \end{pmatrix} = \begin{pmatrix} y - \tau(1 - o(x_o(t-\delta)))y - c(y) \\ y(1-\tau) \end{pmatrix} \tag{10}$$

and the modified replicator dynamic is then given by the system

$$\begin{aligned} \dot{x}_o(t) &= x_o(t)\left(\hat{F}_o(x(t)) - x(t)^T\hat{F}(x(t))\right) \\ \dot{x}_n(t) &= x_n(t)\left(F_n - x(t)^T\hat{F}(x(t))\right). \end{aligned} \tag{11}$$

Introducing a delay causes the population state to oscillate over time. It can be shown that the system over time converges to the Nash equilibrium if the delay is not too large, that is, $\delta < \overline{\delta}$ where

$$\overline{\delta} = \frac{\pi}{2x_o^\star(1 - x_o^\star)\tau y(-o'(x_o^\star))}$$

(see Appendix B). Otherwise, the system doesn't converge and continues to oscillate. In an economic context, this means that if it takes too long to amend the tax code to reflect changed taxpayer behavior, an equilibrium tax law and equilibrium population shares of optimizers and non-optimizers cannot be reached. Figure 2.3 illustrates the population's evolution for different values of delay. If there is no delay, the share of optimizers approaches the Nash equilibrium and remains there. If the delay is small, the oscillation around the equilibrium is dampened after some time. In both cases the tax code is not amended any more once the equilibrium is reached. For delay values that are equal or greater than the critical delay $\overline{\delta}$ the population share of optimizers continues to oscillate. Accordingly, the tax code keeps changing, too. In reality, tax codes are updated on a regular basis. In the context of this model this phenomenon can be explained by real-world governments reacting too slowly

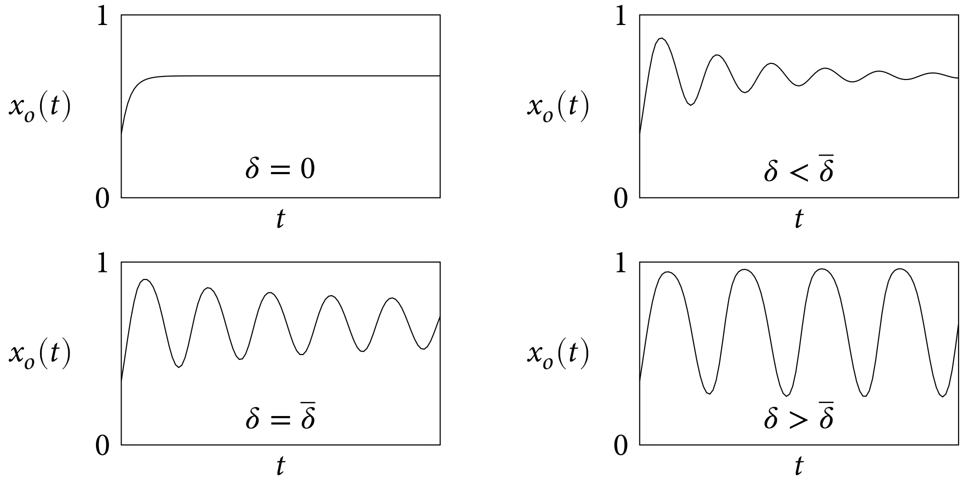

FIGURE 2.3: *Evolution of the share of optimizing taxpayers for different values of delay; the initial population share of optimizers is 0.35.*

to changed taxpayer behavior implying that a stable equilibrium tax law cannot be reached.[12]

3. Extended Model with Tax Evasion

3.1. Framework with Three Strategies

Taxpayers may also attempt to reduce their tax burden in an illegal manner. The economic difference between (illegal) tax evasion and (legal) tax optimization is that the latter involves paying optimization costs in advance, which can be either costs for engaging a professional tax consultant, opportunity costs for having to cope with the tax code, or a disutility from doing so, or a mixture of all three. By contrast, tax evasion does not require an ex ante payment. The taxpayer just reports less income and thus generates tax savings immediately. Afterwards, they are confronted with a certain probability of being audited and having to pay a penalty fee. Many other analytical models of tax evasion assume

[12]Of course, there are many additional *exogenous* reasons why the tax code needs to be amended which are not covered in this model: technological advancements, financial globalization, and so forth. The essential insight in this model is, however, that the tax code can change even *without* such external "shocks".

that the audit probability is exogenous[13] or conditional on reported income[14]. However, I do not assume that the detection probability depends on the *amount* evaded / reported by an individual but on the *proportion* of the population that evades taxes. This assumption is taken in order to fade out the effects of the reported amount in favor of shedding light on the population effects which are of interest in this article. In addition, the tax authority can estimate the population rate of evaders since the latter should be almost identical—but at least highly correlated—with the detection rate. It seems plausible for the tax authority to increase its audit effort when it realizes that tax evasion behavior is starting to spread within society.

The extended set of strategies is denoted by $\bar{S} = \{e, o, n\}$ where e denotes the "tax evasion" strategy. The population state is now given by $\bar{X} = \{x \in \mathbb{R}^3_+ : \sum_{s \in S} x_s = 1\}$. As with tax optimization, the tax authority's reaction to tax evasion is reflected by the payoff function. Audit probability is denoted by the function $p(x_e)$ which strictly increases in the share of evading taxpayers x_e. Further, it is assumed that $p(1) = 1$ and $p(0) = 0$: if the whole population evades it is reasonable for the tax authority to always audit. By contrast, if no one evades, it is rational to never audit. Since the audit probability does not depend on the amount that is evaded, a risk-neutral taxpayer will report "all or nothing". Thus, choosing the evasion-strategy implies that a taxpayer will report an income of zero. An evading taxpayer receives their pre-tax income y if no audit takes place; if audited, they receive their pre-tax income y minus tax payment minus penalty payment. Evading taxes illegally delivers an expected payoff of

$$F_e(x) = p\left(x_e\right)\left(y - \tau y - \theta \tau y\right) + (1 - p(x_e))y, \tag{12}$$

where $\theta > 0$ is the penalty rate. Note that the penalty is imposed on the amount of taxes evaded, τy, as proposed by Yitzhaki (1974). If the whole population evades, the "tax evasion" strategy is always dominated by non-optimization since $\tau(1 + \theta) > \tau$. If no one evades, the "evasion" strategy dominates non-optimization: $y > y(1 - \tau)$. The payoff functions of the "optimization" and "non-optimization" strategies are given by Equations (1) and (2), respectively, giving the new payoff vector field $\bar{F}(x) = (F_e(x), F_o(x), F_n)^T$. The potential

[13] E. g. Allingham and Sandmo (1972), Yitzhaki (1974).
[14] E. g. Reinganum and Wilde (1986), Erard and Feinstein (1994).

function of the extended game is given by

$$\bar{f}(x) = (x_e + x_o + x_n)y - (x_o + x_n)\tau y - x_o c(y) + \tau y \int_0^{x_o} o(z)\,dz$$
$$- y\tau(1+\theta)\int_0^{x_e} p(z)\,dz.$$

Again, $\bar{f}(x)$ is concave,[15] implying that all Nash equilibria are maximizers of $\bar{f}$.

3.2. Equilibrium

3.2.1. Survival of three strategies

Consider first the case where all strategies are played by positive population shares in equilibrium. The only Nash equilibrium is denoted by the population state $\{x_e^*, x_o^*, x_n^*\}$ satisfying the conditions

$$p(x_e^*) - \frac{1}{1+\theta} = 0, \tag{13}$$

$$\tau o(x_o^*) - \frac{c(y)}{y} = 0, \tag{14}$$

$$x_e^* + x_o^* + x_n^* = 1, \tag{15}$$

where (14) is similar to (3). The comparative statics with respect to x_o^* derived from the two strategy game are therefore still valid. Interestingly—in contrast to most other analytical tax compliance models—the share of evading taxpayers does *not* depend on the tax rate. Instead, the only parameter that affects tax evasion behavior is the penalty rate θ. Deriving (13) with respect to θ delivers

$$\frac{\partial x_e^*}{\partial \theta} = -\frac{1}{(1+\theta)^2 p'(x_e^*)} < 0. \tag{16}$$

Increasing the penalty causes taxpayers to switch from tax evasion to non-optimization, ignoring the possibility of legal optimization. If the tax authority

[15] The hessian $H_{\bar{f}}(x) = \begin{pmatrix} -y(1+\theta)\tau p'(x_e) & 0 & 0 \\ 0 & y\tau o'(x_o) & 0 \\ 0 & 0 & 0 \end{pmatrix}$ has non-positive eigenvalues $\{0, \tau y o'(x_o), -y(1+\theta)\tau p'(x_e)\}$.

were to introduce a new audit function $p_H(x_e) > p(x_e) \forall x_e \neq 0, 1$ the share of evading taxpayers would decrease and vice versa.

In summary, increasing optimization costs cause the share of optimizers to shrink while the share of non-optimizers increases. By contrast, a higher tax rate causes taxpayers to switch from non-optimization to legal optimization behavior. If income y changes the shift takes place between the share of non-optimizing and optimizing taxpayers; the direction is, however, not distinct. Increasing y may cause optimization activities to increase to the detriment of non-optimization (see Equation (5)). Still, this would not yet be a theoretical explanation for anecdotical evidence that the "rich" are more involved in legal tax optimization than the "poor", since in this model all taxpayers are identical. If income rises, then it rises by the same amount for all taxpayers. Instead, the reason for this effect would be the concave optimization cost structure.

In equilibrium, the tax code allows for tax savings according to Equation (7) and audits will happen with probability

$$p(x_e^*) = \frac{1}{1+\theta}. \tag{17}$$

The tax authority's total tax revenue

$$\begin{aligned}\bar{T} &= \sum_{i \in P : s=n} \tau y + \sum_{i \in P : s=o} \tau y(1 - o(x_o^*)) + \sum_{i \in P : s=e} p(x_e^*)\tau y(1+\theta) \\ &= N(\tau y - x_o^* c(y))\end{aligned}$$

is identical to (8) since the share of optimizing taxpayers is not altered by introducing the possibility to evade taxes. Further, the tax authority loses nothing through tax evasion: in equilibrium, tax evasion is not beneficial to taxpayers (again, by definition of the Nash equilibrium) because expected tax savings equate expected penalty payments. This result of course no longer holds if one assumes audits to be costly to the tax authority, or if penalty payments are not part of tax revenue.

3.2.2. Extinction of Non-Optmization

Depending on the choice of parameters, the "non-optimization" strategy may become extinct over time. This is the case if the payoff of either "evasion" or "optimization" in equilibrium exceeds the payoff of "non-optimization". The section above demonstrates that an increase in the shares of both evaders (possibly caused by a decrease in the penalty rate) and optimizers (possibly caused by an increase in the tax rate) goes entirely to the detriment of the share of non-optimizing taxpayers. If x_n reaches zero it cannot decrease any further. Hence, if the share of evaders (optimizers) were to increase to a greater degree, then the share of optimizers (evaders) would decrease, respectively. This would cause the payoff of the "optimization" ("evasion") strategy to increase. In equilibrium these two payoffs have to balance again, that is, $F_e(x) = F_o(x) > F_n$. This equilibrium is denoted by the population state $\left\{x_e^\dagger, x_o^\dagger, 0\right\}$ that fulfills the conditions

$$\tau y(1+\theta)p(x_e^\dagger) = \tau y + \tau y o(x_o^\dagger) - c(y), \tag{18}$$

$$x_e^\dagger + x_o^\dagger = 1. \tag{19}$$

This is the Nash equilibrium if only "evasion" and "optimization" are chosen by positive population shares.[16] The condition for non-optimization to become extinct can be derived from either one of the equivalent conditions

$$F_e(x_e^\dagger) \geq F_n \iff \theta \leq \frac{1-p(x_e^\dagger)}{p(x_e^\dagger)}, \tag{20}$$

$$F_o(x_o^\dagger) \geq F_n \iff c(y) \leq \tau y o(x_o^\dagger). \tag{21}$$

Deriving (18) with respect to y, making use of the fact that $\frac{\partial x_e^\dagger}{\partial y} + \frac{\partial x_o^\dagger}{\partial y} = 0$ and rearranging delivers

$$-\frac{\partial x_e^\dagger}{\partial y} = \frac{\partial x_o^\dagger}{\partial y} = \frac{c(y) - yc'(y)}{\tau y^2 \left(-o'\left(x_o^\dagger\right) + (1+\theta)p'\left(x_e^\dagger\right)\right)},$$

the sign of which is identical to the comparable derivative in the two-strategy game $\frac{\partial x_o^\star}{\partial y}$ (Equation 5). It is positive if average costs exceed marginal costs, if costs decrease with income or if costs are constant. Increasing income then causes the population share of optimizers to increase to the detriment of the population share of evaders. Deriving (18) with respect to τ and rearranging

[16] Of course, it would also be the Nash equilibrium of a model which requires taxpayers to choose between optimization and evasion only.

gives

$$-\frac{\partial x_e^\dagger}{\partial \tau} = \frac{\partial x_o^\dagger}{\partial \tau} = \frac{c(y)}{\tau^2 y \left(-o'\left(x_o^\dagger\right) + (1+\theta)p'\left(x_e^\dagger\right)\right)},$$

which is positive. Increasing the tax rate causes the population share of optimizers to increase. This effect can be explained as follows: since this model employs the penalty structure of Yitzhaki (1974) there is no substitution effect in the "evasion" strategy. As taxpayers are assumed to be risk-neutral, neither is an income effect. Since the optimization cost does not depend on the tax rate, however, optimization becomes more beneficial with increasing tax rates. Finally, the derivative with respect to θ,

$$-\frac{\partial x_e^\dagger}{\partial \theta} = \frac{\partial x_o^\dagger}{\partial \theta} = \frac{p\left(x_e^\dagger\right)}{-o'\left(x_o^\dagger\right) + (1+\theta)p'\left(x_e^\dagger\right)},$$

is positive. Not surprisingly, increasing the penalty rate causes taxpayers to switch to legal optimization.

It is not possible to give a closed-form solution for both $p(x_e^\dagger)$ and $o(x_o^\dagger)$. Hence, the government's total tax revenue cannot be stated explicitly either. However, since the taxpayers' equilibrium payoffs increase, the government's total tax revenue has to be smaller than above. Formally,

$$\begin{aligned} \bar{T} &= \sum_{i \in P : s=o} \tau y(1 - o(x_o^\dagger)) + \sum_{i \in P : s=e} p(x_e^\dagger)\tau y(1+\theta) \\ &= n\left(\tau y - x_o^\dagger c(y) + c(y) - \tau y o\left(x_o^\dagger\right)\right). \end{aligned} \tag{22}$$

Equations (18) and (19) are used. Comparing (22) with (8) one finds that the former is smaller than the latter if

$$c(y)\left(1 - x_o^\dagger + x_o^*\right) < \tau y o\left(x_o^\dagger\right).$$

Comparing (18) and (14) one ascertains that $o\left(x_o^\dagger\right) < o\left(x_o^*\right)$ if (20) holds strictly, hence $x_o^\dagger > x_o^*$. Thus, $(1 - x_o^\dagger + x_o^*) < 1$. If Condition (21) holds strictly, than the inequality is fulfilled. That is, tax revenue decreases if the "non-optimization" strategy is not played.

3.3. Replicator Dynamics with Three Strategies

The model of individual taxpayer's behavior described in section 2.3 is applied to the three strategy-case: upon receiving an opportunity to update their strategy, a taxpayer meets another taxpayer at random and compares payoffs. As with tax evasion, however, recall that (12) is an *expected* payoff. At the micro-level, in contrast, there are two distinct types of tax evaders: those who where audited, incurred a punishment and thus "lost", receiving a payoff of $F_e^L = y(1-\tau(1+\theta))$, and those who "won", receiving $F_e^W = y$. Applying the proportional imitation protocol having regard to these additional instances, weighting each with the probabilities $p(x_e)$ and $(1 - p(x_e))$, respectively, the new replicator dynamic with three strategies is given by the system

$$\begin{array}{rcl} \dot{x}_e & = & x_e\left(F_e(x) - x^T\bar{F}(x)\right) \\ \dot{x}_o & = & x_o\left(F_o(x) - x^T\bar{F}(x)\right) \\ \dot{x}_n & = & x_n\left(F_n - x^T\bar{F}(x)\right) \end{array} \tag{23}$$

which has seven fixed points.[17] Three of them,

$$\begin{array}{l} \{x_e = 1, x_o = 0, x_n = 0\}, \\ \{x_e = 0, x_o = 1, x_n = 0\}, \\ \{x_e = 0, x_o = 0, x_n = 1\}, \end{array}$$

are corner solutions in which the whole population either evades, optimizes or non-optimizes, respectively. It turns out that all corner solutions are unstable source nodes. Both alternative strategies deliver excess return; thus, the population shares of taxpayers playing these strategies would increase to their equilibrium values x_e^* and x_o^* once some taxpayers started to play these strategies. Then there is one fixed point,

$$\{x_e = 0, x_o = x_o^*, x_n = 1 - x_o^*\},$$

in which the population share x_o^* optimizes and the rest of the population non-optimizes with no one evading. This is the Nash equilibrium of the two-strategy game elaborated on in Section 2. In the three-strategy model it is an unstable saddle point: excess return could be generated by evading taxes. Thus, x_e would

[17] A stability analysis is given in Appendix C.1.

reach x_e^* once a single taxpayer started tax evasion. Another fixed point,

$$\{x_e = x_e^*, x_o = 0, x_n = 1 - x_e^*\},$$

is given for a population share of x_e^* evading taxes and the rest non-optimizing with no one optimizing. This would be the Nash equilibrium of a two-strategy model without the possibility to "optimize", but the strategies "evasion" and "non-optimization" only. This fixed point is also an unstable saddle point in the three-strategy model because optimizing delivers excess return. If no one non-optimizes the fixed point

$$\left\{x_e = x_e^\dagger, x_o = x_o^\dagger, x_n = 0\right\}$$

can be reached. This fixed point is stable if Conditions (20) and (21) hold, that is, if the parameters are such the "non-optimization" strategy in equilibrium delivers a payoff that is worse than "evasion" and "optimization", and thus "non-optimization" becomes extinct. Otherwise, the fixed point is an unstable saddle point because "non-optimization" delivers excess return. Finally, the Nash equilibrium

$$\{x_e = x_e^*, x_o = x_o^*, x_n = 1 - x_o^* - x_e^*\}$$

elaborated on above is a stable fixed point of the system (23). See Appendix C.1 for a detailed stability analysis of all fixed points. Figure 3.1 shows the flow pattern in the three-strategy space. At the corners of the simplex the whole population either evades, optimizes or non-optimizes, respectively. The respective corner solutions are source nodes: all arrows point away from the corners. All population states on the edges require the strategy of the opposite corner to be non-existent within the population. In this example the Nash equilibrium requires all three strategies to be played by positive population shares. Thus, all rest points on the edges are saddle points. The arrows point from the corners towards those rest points; however, they are unstable. If the third strategy is played by a single individual the population state moves towards the Nash equilibrium indicated by a black circle in the bottom-left corner of the simplex, which involves all strategies to be played by a fraction of all individuals.

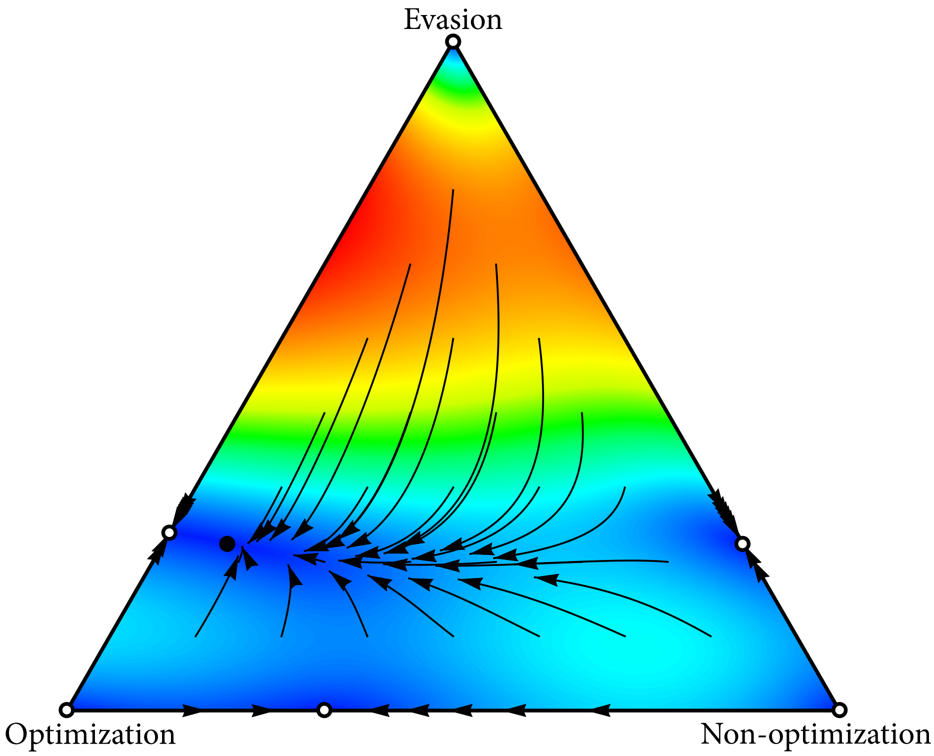

Figure 3.1: *Flow pattern for the three-strategy game. Red (blue) colors indicate fast (slow) movement. White circles indicate the unstable rest points; the black circle shows the stable rest point. The figure was created using the Mathematica application "Dynamo" by Sandholm, Dokumaci, and Franchetti (2012).*

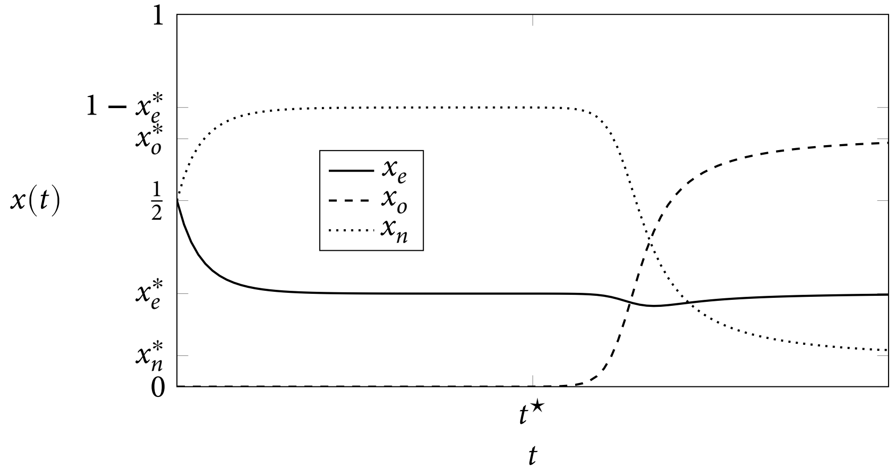

FIGURE 3.2: *Evolution of the population state over time for 4 000 individuals. The initial population state is $\frac{1}{2}$ evaders and $\frac{1}{2}$ non-optimizing taxpayers. At time $t^\star$ the optimization strategy is invented by one formerly non-optimizing taxpayer.*

While the saddle point on the right-hand edge of Figure 3.1—where no one optimizes—is not stable if the "optimization" strategy is available, it would be stable if such an alternative did not exist.

The "optimization" strategy can also be interpreted in a more specific way as a particular (legal) tax avoidance model rather than general tax planning. A popular example of such a model is the Double Irish arrangement. The former tax law reaction function $o(x_o)$ could then be interpreted as the probability of such a model being accepted by the tax authority. The assumption that $o(x_o)$ is strictly decreasing would still be reasonable: the more prominent such a model becomes, the higher the probability that it will be rejected by the tax authority or a fiscal court. F_o would then be the expected value of the (uncertain) income after tax. The model can then capture the behavior of the population if a specific tax savings model is invented by a single taxpayer at time $t^\star$. Figure 3.2 illustrates the behavior of the system over time for a population of 4 000 taxpayers.

Starting from an initial population state of $1/2$ evaders and $1/2$ non-optimizing taxpayers the system approaches the rest point with x_e^* evaders and

$1 - x_e^*$ non-optimizing individuals and remains there. At time $t^\star$ one formerly non-optimizing taxpayer develops a particular tax optimization model. Since the initial population share of optimizers is only $1/4\,000$ some time elapses until the strategy starts to spread to other parts of the population. Then, x_o increases quickly and the population state approaches the Nash equilibrium of the three-strategy game. The equilibrium population share of evaders is the same in both the two-strategy (evasion and non-optimization only) and the three-strategy game. There is just a small dent in x_e shortly after the invention of the optimization model during the adjustment process towards the new equilibrium. The equilibrium share of optimizers goes entirely to the detriment of the share of non-optimizing taxpayers.

3.4. Delayed Amendments and Tax Evasion

In Section 2.4 it was argued that the tax authority may not be able to amend the tax code immediately. Thus, a delay δ is introduced to the "optimization" strategy in the extended model, too. Of course, the tax authority may not be able to adjust the audit rate immediately as well, since the population share of evaders is not known before actually having audited the population. On the other hand, adjusting the audit rate is far easier than amending the code. If the tax authority, while auditing, realizes that a lot of tax reports are incorrect it could immediately decide to broaden its audit activities, possibly following a Bayesian updating inference. Therefore in this section it is assumed that the audit rate can be adjusted instantly whereas the tax code is adjusted with delay. This gives the new payoff vector field $\bar{F}_\delta = \left(F_e(x), \hat{F}_o(x), F_n\right)^T$ where $\hat{F}_o(x)$ includes the delay δ as stated in Equation (10). The replicator dynamic including delay is given by the system

$$\begin{aligned} \dot{x}_e &= x_e\left(F_e - x^T \bar{F}_\delta(x)\right) \\ \dot{x}_o &= x_o\left(\hat{F}_o - x^T \bar{F}_\delta(x)\right) \\ \dot{x}_n &= x_n\left(F_n - x^T \bar{F}_\delta(x)\right). \end{aligned} \tag{24}$$

Again, introducing a delay to the "optimization" strategy causes the population state to oscillate over time. The analysis is carried out for the case that in equilibrium all three strategies are played by positive population shares. I. e., it is assumed that the Conditions (20) and (21) do not hold. For zero delay the system is stable at x^*. It turns out that the system remains stable for all $\delta < \hat{\delta}$,

$$\hat{\delta} = \frac{1}{\omega_+} \operatorname{arccot}\left(\frac{\omega_+(d - al)}{ad + l\omega_+^2}\right), \tag{25}$$

where

$$\begin{aligned} a &= (1 - x_e^*)\, x_e^* \tau(1 + \theta) y p'\,(x_e^*) > 0, \\ l &= (1 - x_o^*)\, x_o^* \tau y\,(-o'\,(x_o^*)) > 0, \\ d &= x_e^* x_o^*\,(1 - x_e^* - x_o^*)\,(1 + \theta)\tau^2 y^2\,(-o'\,(x_o^*))\, p'\,(x_e^*) > 0, \\ \omega_+ &= \left(\frac{1}{2}\left((l^2 - a^2) + \sqrt{(l^2 - a^2)^2 + 4d^2}\right)\right)^{1/2} > 0. \end{aligned}$$

For higher delay values $\delta > \hat{\delta}$ the system becomes unstable and stability cannot be regained with increasing delay further. Refer to Appendix C.2 for details. Figure 3.3 shows the evolution of the population state for different values of delay. While the main shift takes place between optimizers and non-optimizers, the adaption process causes the share of evaders to oscillate, as well. Whereas the state of the tax law is hard to measure empirically, the fraction of tax evaders is easy to assess. Thus, it is an empirically testable hypothesis whether tax evasion rates oscillate over time. In the sense of this model, such a finding could be an indication that (hard-to-observe) optimization activities and the (almost unobservable) state of the tax law also change over time.

4. Conclusion

This article aims to contribute to the scarce theoretical literature on the strategic interdependency between taxpayers and tax authority relating to legal tax avoidance. It is assumed that taxpayers can legally avoid taxes by searching for appropriate legal norms in the tax code, which is associated with optimization costs. The tax authority reacts by closing certain loopholes if they are exploited by too many taxpayers. It emerges that the share of optimizing taxpayers increases if the tax rate increases, optimization costs decrease and tax law gets less tight. If the legislation reacts to changed taxpayer behavior with a delay, the population shares of optimizers and the tax law oscillate over time. It is shown that the tax law can change *endogenously* by explicitly modeling

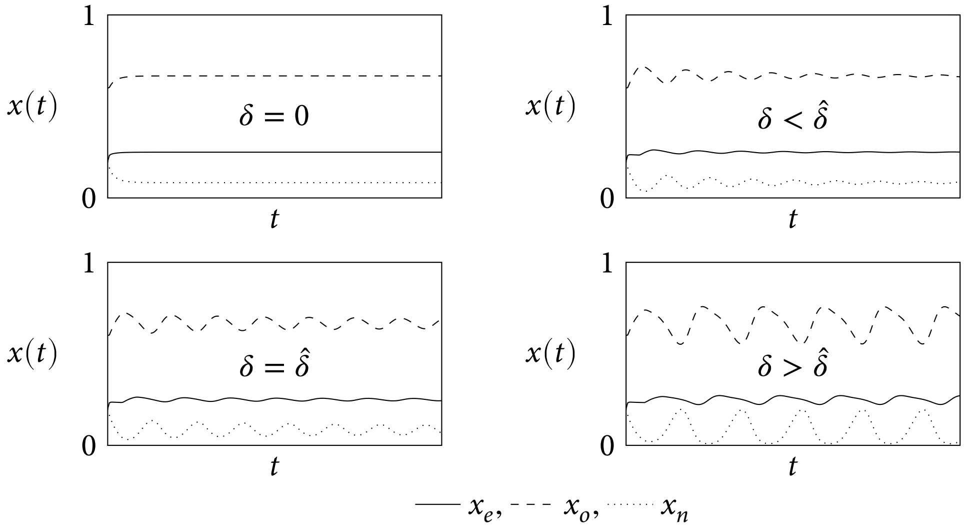

FIGURE 3.3: *Evolution of the population state for different values of delay.*

the adaption process towards equilibrium. If the delay is not too large, the oscillation is dampened over time. Otherwise, however, the population state keeps oscillating and the Nash equilibrium cannot be reached. As a policy implication, it is recommended to accelerate the legislation process in order to avoid costly repeated amendments to the tax code.

Then, tax evasion is introduced as a third strategy. Depending on parameter values, there exist two Nash equilibria. The first equilibrium consists of all three strategies being played by positive population shares. It turns out that the population shares of optimizers and evaders are not interdependent; instead an increase in both groups goes fully to the detriment of the share of non-optimizing taxpayers. A second equilibrium is reached if the payoff to non-optimization is worse than the payoffs of both optimization and evasion. Then, no one applies the "non-optimization"-strategy. The share of optimizers increases to the detriment of the share of evaders if the tax rate or the penalty rate increase. Introducing a delay again causes the population shares and the tax law to oscillate over time. The oscillation is not dampened if the delay exceeds a certain threshold. This confirms the policy implication found above.

Of course, there are several limitations to the model. First, if the crowding effect in the "optimization"-strategy is interpreted as tax complexity, the fact is neglected that increasing tax complexity also affects the payoffs of non-optimizing taxpayers and of audited evading taxpayers since they need to spend more resources on coping with a complex tax code. This holds no longer, however, if the reduction in tax savings after an increase in the share of optimizers is interpreted as anti-tax avoidance doctrines or legal norms that reduce the profitability of certain tax shelters but do not bother individuals who pay their taxes regularly. The assumption that the tax law reaction function is continuous is also a simplified one. A more realistic tax law reaction function would be likely to jump if the number of optimizers exceeds a certain threshold. Future research could implement a discontinuous tax law reaction function. Then, the assumptions that tax savings possibilities decrease if more taxpayers optimize and that audit probability increases if more taxpayers evade taxes are not empirically tested. However, neither is it certain that the crucial assumption of some other tax compliance models is valid, namely that audit probability depends on the amount of reported income. Both assumptions taken in this model seem plausible; whether or not they are true is ultimately an empirical question.

The main results of this paper could also be empirically tested: increasing the tax rate ought to leave the extent of tax evasion unaffected; instead, optimization activities—which could be measured by offsets of tax consultancy costs—ought to increase. While the state of the tax law is hard to measure, amendments to the tax code are easy to observe. It can also be empirically tested if audit rates vary over time, which could be an indirect indication of a changing tax law.

Future research should allow taxpayers to be heterogenous with respect to their income and possibly other individual characteristics, such as risk aversion. Also, a social disutility from behaving "immorally" could be integrated. Finally, it would be desirable to drop the mean field assumption and instead to incorporate a social network model.

Appendix

A. Exemplary Reaction Functions and Parameters

The reaction functions and parameters used for illustration purposes in this article are chosen according to the following table.

Pre-tax income	y	10
Tax rate	τ	0.3
Penalty rate	θ	3
Optimization cost function	$c(y)$	$0.1y$
Tax law reaction function	$o(x_o)$	$1 - x_o$
Audit reaction function	$p(x_e)$	x_e

TABLE A.1: *Reaction functions and parameters used in the illustrations.*

B. Stability of the Replicator Dynamic with Two Strategies and Delay

Since $x_n = 1 - x_o$ it suffices to study the stability of $\dot{x}_o$. Let $z(t) = x_o(t) - x_o^\star$. The linear variational system of (11) is then

$$z'(t) + x_o^\star(1 - x_o^\star)\tau y(-o'(x_o^\star))z(t - \delta) = 0. \tag{26}$$

The fixed point $\{x_n = x_n^\star, x_o = x_o^\star\}$ is asymptotically stable for the system (11) if the trivial solution of (26) is asymptotically stable (Bellman & Cooke, 1963, p. 336). As shown by Freedman and Kuang (1991, p. 195), (26) is stable if $\delta < \overline{\delta}$ where

$$\overline{\delta} = \frac{\pi}{2x_o^\star(1 - x_o^\star)\tau y(-o'(x_o^\star))}.$$

C. Replicator Dynamic with Three Strategies

C.1. Stability without Delay

To evaluate the stability of the fixed points of the system (23) it suffices to study the system of two equations, $\dot{x}_e$ and $\dot{x}_o$ only, where $x_n = 1 - x_e - x_o$. Since $\dot{x}_n = \dot{x}_e - \dot{x}_o$ it must be that $\dot{x}_n$ is stable whenever $\dot{x}_e$ and $\dot{x}_o$ are stable. The eigenvalues of the Jacobian $J(x_e, x_o)$ of the system are examined at the respective fixed points (Bellman & Cooke, 1963, p. 338).

The first fixed point requires all taxpayers to evade. The eigenvalues of $J(1, 0)$ are given by $\{\theta\tau y, (1 + \theta)\tau y - c(y)\}$. Since $\tau y > c(y)$ by assumption all eigenvalues are positive; the first fixed point is an unstable source node.

The second fixed point is reached if all taxpayers optimize. It is also a source node since the eigenvalues of $J(0,1)$, $\{c(y), \tau y + c(y)\}$, are positive.

The third corner solution requires all taxpayers to non-optimize: the shares of evaders and optimizers are zero. The eigenvalues of $J(0,0)$ are given by $\{\tau y, \tau y - c(y)\}$. Since they are positive, the population state with all taxpayers non-optimizing is an unstable source node, as well.

The next fixed point requires the share x_o^* to optimize whereas the remainder, $1 - x_o^*$, non-optimizes with no one evading. The eigenvalues of $J(0, x_o^*)$ are given by $\{\tau y, \tau\left(1 - x_o^*\right) x_o^* y o'\left(x_o^*\right)\}$. The first eigenvalue is positive, the second is negative; the fourth fixed point is thus an unstable saddle point.

Another fixed point is given if the share x_e^* evades and the remainder, $1 - x_e^*$, non-optimizes. The eigenvalues of $J(x_e^*, 0)$ are given by $\{\tau y - c(y), -(1 + \theta)\tau y\left(1 - x_e^*\right) x_e^* p'\left(x_e^*\right)\}$. Again, the eigenvalues have opposite signs. The fifth fixed point is thus also a saddle point.

If no one non-optimizes, a fixed point can be reached in which the share $x_e^\dagger$ evades and the share $x_o^\dagger = 1 - x_e^\dagger$ optimizes. The eigenvalues of $J(x_e^\dagger, x_o^\dagger)$ are given by

$$\begin{aligned}&\left\{\tau y\left((1+\theta)p\left(x_e^\dagger\right) - 1\right),\right.\\ &\left.\tau y\left(1 - x_e^\dagger\right) x_e^\dagger\left(o'\left(1 - x_e^\dagger\right) - (1+\theta)p'\left(x_e^\dagger\right)\right)\right\}.\end{aligned}$$

The second eigenvalue is always negative. The first is negative if the condition (20) holds strictly. The fixed point is then a stable sink node; it is an unstable saddle point otherwise.

Finally, the eigenvalues of $J(x_e^*, x_o^*)$ are given by $\{\psi - \sqrt{\varphi}, \psi + \sqrt{\varphi}\}$, where

$$\psi = -\frac{1}{2}\tau y\left((1 - x_e^*)x_e^*(1+\theta)p'(x_e^*) + (1 - x_o^*)x_o^*(-o'(x_o^*))\right)$$

is clearly negative and the sign of

$$\begin{aligned}\varphi =&\tau^2 y^2 \frac{1}{4}\left(\left(-(1 - x_o^*)x_o^* o'(x_o^*) + (1 - x_e^*)x_e^*(1+\theta)p'(x_e^*)\right)^2\right.\\ &\left.-4x_e^* x_o^*(1 - x_e^* - x_o^*)(1+\theta)\left(-o'(x_o^*)\right)p'(x_e^*)\right)\end{aligned}$$

is not distinct. If φ is positive then $\psi - \sqrt{\varphi}$ is always negative. Further, $\psi + \sqrt{\varphi}$ is negative if $\sqrt{\varphi} < -\psi \Leftrightarrow \varphi < \psi^2$. This gives the condition

$$x_e^*(1+\theta)x_o^*\tau^2 y^2(1 - x_e^* - x_o^*)(-o'(o))p'(e) > 0$$

which is always fulfilled. It can be concluded that the fixed point is stable if φ is positive, that is, if the eigenvalues of $J(x_e^*, x_o^*)$ are real. If φ is negative the eigenvalues take the form $\{\psi - i\sqrt{-\varphi}, \psi + i\sqrt{-\varphi}\}$. Since the eigenvalues have negative real parts the system behaves as a damped oscillator; the fixed point is also stable.

C.2. Stability with Delay

Again, only the stability of $\dot{x}_e$ and $\dot{x}_o$ needs to be studied. Let $u(t) = x_e(t) - x_e^*$ and $v(t) = x_o(t) - x_o^*$. The variational system of (24) about x^* is given by

$$\begin{aligned} u'(t) &= -au(t) + bv(t-\delta) \\ v'(t) &= ku(t) - lv(t-\delta) \end{aligned} \tag{27}$$

(Bellman & Cooke, 1963, p. 339), where

$$\begin{aligned} a &= (1 - x_e^*)\, x_e^* \tau(1+\theta) y p'\,(x_e^*) > 0, \\ b &= x_e^* x_o^* \tau y\,(-o'\,(x_o^*)) > 0, \\ k &= x_e^* x_o^* \tau(1+\theta) y p'\,(x_e^*) > 0, \\ l &= (1 - x_o^*)\, x_o^* \tau y\,(-o'\,(x_o^*)) > 0. \end{aligned}$$

The system (27) can be written as

$$v''(t) + av'(t) + lv'(t-\delta) + dv(t-\delta) = 0 \tag{28}$$

where $d = (al - kb)$. The Laplace transform of (28) is given by

$$\lambda^2 + a\lambda + l\lambda e^{-\lambda\delta} + de^{-\lambda\delta} = 0. \tag{29}$$

The roots of (29) are given by $\lambda = i\omega, \omega > 0$. From equation (4.6) in Freedman and Kuang (1991, p. 199) one has

$$\omega_{\pm}^2 = \frac{1}{2}\left((l^2 - a^2) \pm \sqrt{(l^2 - a^2)^2 + 4d^2}\right). \tag{30}$$

Theorem 4.1 of Freedman and Kuang (1991, p. 202) is applied. $d \neq 0$ if

$$(\tau y - c(y))(\tau y - (1+\theta)c(y)) \neq 0.$$

$\tau y > c(y)$ by an assumption made in Section 2.1. The additional assumption $\tau y \neq (1+\theta)c(y)$ has to be made. Since $0 < d^2$ only one imaginary root exists (Freedman & Kuang, 1991, p. 200). Hence the system is stable if $\delta < \hat{\delta}$, and unstable afterwards, with

$$\hat{\delta} = \frac{\eta}{\omega_+}, \tag{31}$$

where

$$\cos\eta = -\frac{al\omega_+^2 - d\omega_+^2}{l^2\omega_+^2 + d^2}, \tag{32}$$

$$\sin\eta = \frac{da\omega_+ + l\omega_+^3}{l^2\omega_+^2 + d^2} \tag{33}$$

according to equations (4.13) and (4.14) of Freedman and Kuang (1991, p. 201); thus,

$$\eta = \operatorname{arccot}\left(\frac{\omega_+(d - al)}{ad + l\omega_+^2}\right). \tag{34}$$

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